

# Problem G All Are Equal

Joni inherited a large vacant land from his billionaire grandfather. As a young and ambitious but nature-loving man, he decided to build a park with a lot of trees in his vacant land. He has bought N trees and planted them in a single long line. The trees are numbered from 1 to N, sequentially. Why a single long line? We also don't understand, he said that it is beautiful to arrange things in a single line, presumably due to his weird sense of art.

Later Joni noticed that all those trees are not necessarily on the same height. In his frustration, he installed Equalator, a "magical" machine he borrowed from Dr. Doofenshmirtz, next to tree number 1. This device is able to increase or decrease the height of one or more trees. Specifically, this device only accepts a work order represented by two integers (R, H), which means, it will increase (or decrease, if H is negative) the height of **all trees** between tree number 1 and tree number R, inclusively, by H. At any moment, the height of any trees should be a positive integer; in other words, it will not work (a.k.a. explode!) if the given work order will cause the height of any tree becomes zero or negative.

Equalator runs on special mineral which is hard to obtain, thus Joni needs to give a minimum number of work order to Equalator to achieve his goal, i.e. all trees are on the same height. Let the number of work order given to the Equalator be W, the cost to run Equalator is  $W\sqrt{W}$  or simply  $W^{1.5}$ .

For example, let N = 3 and the heights are  $H_{1..3} = \{4, 6, 3\}$ . Then, Joni needs to give at least two work orders to accomplish his goal:

- (2, -3), decrease the height of the 1<sup>st</sup> and 2<sup>nd</sup> tree by 3  $\rightarrow$  H<sub>1..3</sub> = {1, 3, 3}.
- (1, 2), increase the height of the 1<sup>st</sup> tree by 2  $\rightarrow$  H<sub>1..3</sub> = {3, 3, 3}.

Therefore, the minimum total cost for this example will be  $2^{1.5} = 2.828427124746...$ 

However, the problem is more complicated than he thought. Joni does not know the initial height of all trees he has planted. The possible height of the i<sup>th</sup> tree, H<sub>i</sub>, is an integer which is distributed uniformly at random between  $L_i$  and  $R_i$  (inclusive). While waiting for him to find out the height of each tree, Joni asked you to calculate the expected minimum cost required to accomplish his goal.

For example, let N = 2, and the heights' range are  $L_{1..2} = \{1, 1\}$  and  $R_{1..2} = \{2, 2\}$ ; it means there are two trees and each tree's height is between 1 and 2. There are four possible cases:

- $H_{1..2} = \{1, 1\} no work order is needed.$
- $H_{1..2} = \{1, 2\} a$  minimum of one work order is needed: (1, 1).
- $H_{1..2} = \{2, 1\} a$  minimum of one work order is needed: (1, -1).
- $H_{1..2} = \{2, 2\} no work order is needed.$

The expected minimum total cost for this example is:  $(0^{1.5} + 1^{1.5} + 1^{1.5} + 0^{1.5}) / 4 = 2 / 4 = 0.5$ 

Given the height range for each tree, determine the expected minimum total cost to run the Equalator in order to achieve Joni's goal.



### Input

The first line of input contains an integer T ( $T \le 100$ ) denoting the number of cases. Each case begins with an integer N ( $1 \le N \le 100$ ) in a line denoting the number of trees. The next N lines, each contains two integers:  $L_i R_i$  ( $1 \le L_i \le R_i \le 1,000,000$ ), which means the height range of the i<sup>th</sup> tree is between  $L_i$  and  $R_i$ , inclusive.

## Output

For each case, output "Case #x: Y" (without quotes) in a line where x is the case number (starts from 1), and Y is the answer for this particular case. Your answer will be considered correct if the relative or absolute difference between your answer and judge's answer is not more than 10<sup>-8</sup>.

Sample Input	Output for Sample Input
3	Case #1: 0.5000000
2	Case #2: 2.82842712
1 2	Case #3: 1.44280904
1 2	
3	
4 4	
6 6	
3 3	
3	
1 1	
1 2	
1 3	

#### Explanation for 1<sup>st</sup> sample case

This is the (second) example given in the problem statement.

#### Explanation for 2<sup>nd</sup> sample case

This is the (first) example given in the problem statement; in this example, all heights are precisely known.

#### Explanation for 3rd sample case

There are six possibilities in this case:

- $H_{1..3} = \{1, 1, 1\}$  no work order is needed.
- $H_{1..3} = \{1, 1, 2\} a$  minimum of one work order is needed: (2, 1).
- H<sub>1..3</sub> = {1, 1, 3} a minimum of one work order is needed: (2, 2).
- H<sub>1..3</sub> = {1, 2, 1} a minimum of two work orders are needed: (1, 1), (2, -1).
- $H_{1..3} = \{1, 2, 2\} a$  minimum of one work order is needed: (1, 1).
- H<sub>1..3</sub> = {1, 2, 3} a minimum of two work orders are needed: (2, 1), (1, 1).

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The expected cost is:
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- $= (0^{1.5} + 1^{1.5} + 1^{1.5} + 2^{1.5} + 1^{1.5} + 2^{1.5}) / 6$
- = 8.65685424949... / 6
- = 1.44280904158...